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Absorption of thermal radiation in a semi-transparent spherical droplet: a simplified model

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Abstract

The boundary-value problem for calculation of differential absorption of thermal radiation is formulated based on the modified DP_0 approximation. The solution of this problem is supplemented by simple analytical approximations for the normalised absorbed radiation power. The latter is used together with the analytical approximation for the efficiency factor of absorption, suggested earlier. The resulting simplified model is applied to the specific problem of absorption of thermal radiation by a diesel fuel droplet. Two types of diesel fuel have been considered. It is pointed out that the radial distribution of absorbed thermal radiation power is non-monotonic. The power absorbed in the droplet core is shown to be rather large and almost homogeneous. Also, the absorbed power is large in the vicinity of the droplet surface, but is minimal in the intermediate region. It is pointed out that the variations of the refractive index of diesel fuel with wavelengths can smooth the predicted radial dependence of the thermal radiation power, absorbed in diesel fuel droplets.

Keywords: Droplet; Thermal radiation; Absorption; Diesel engines; Fuel

1. Introduction

Bertoli and Migliaccio (1999) drew attention to the fact that taking into account the temperature gradient inside fuel droplets would increase the accuracy of computation of the combustion processes in diesel engines. However, the model used by these authors did not account for the effect of thermal radiation. Accurate modelling of the process of droplet heating requires not only the knowledge of the total radiative power absorbed by the droplets, but also the distribution of the power absorbed within the droplets. This paper will be focused on the latter problem.

The radii of diesel fuel droplets r_d are generally much larger than the wavelengths λ of thermal radiation (Pitcher et al., 1990; Comer et al., 1999; Sirignano, 1999). This allows us to ignore the wave effects and use the geometrical optics approximation. The range of applicability of this approximation was investigated by a number of authors (e.g. Lage and Rangel, 1993; Velesco et al., 1997; Dombrovsky, 2000). Direct comparison of results of calculations based on the Mie theory and on the geometrical optics approximation have shown that the latter is applicable for droplets with the diffraction parameter $x = 2\pi r_d/\lambda$ greater than 20 (Dombrovsky, 2000). Although the calculations based on the geometrical optics approximation are much simpler than those based on the Mie theory, they still seem to be too complicated for engineering applications. A new approach, called modified DP_0 (MDP_0) approximation, led to a reduction of the computational time by two orders of magnitude, while maintaining the required accuracy of results (Dombrovsky, 1998, 2000, 2002a). Despite its simplicity, the solution of the problem based on the MDP_0 approximation still requires numerical solution of the boundary-value problem. This fact and relatively small contribution of thermal radiation to the

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overall droplet heating make the direct application of this approximation in multidimensional CFD codes unpractical at the moment.

In this paper, a further simplification of the model for the differential heating of a fuel droplet by thermal radiation is suggested. Simplicity of this approach makes it potentially attractive for implementation into the above mentioned multidimensional CFD codes, while its accuracy is sufficient for most practical engineering applications, including those in diesel engines (Aggarwal, 1998; Sazhina et al., 2000). Following Harpole (1980) and Dombrovsky (2000), spherical symmetry of the distribution of thermal radiation inside the droplet and at its boundary is assumed. This assumption leads to considerable simplification of the solution of the radiation transfer problem compared with the general solution considered by Lage and Rangel (1993). Note that the irradiation of fuel droplets in diesel engines is, strictly speaking, not symmetric (e.g. Mengüç et al., 1985; Viskanta and Mengüç, 1987). Intensive scattering of thermal radiation by fuel droplets and large optical thickness of fuel sprays, however, is expected to lead to the situation when this irradiation of droplets inside the spray is close to being spherically symmetric (Dombrovsky, 1996, 2002b). The irradiation of droplets at the periphery of the spray is likely to remain asymmetric. The model developed in this study is not applicable to this case. The general approach, similar to that described by Lage and Rangel (1993), needs to be used. Some preliminary results referring to the topic of this paper have been reported by Dombrovsky and Sazhin (2003).

Basic equations of the model are presented and discussed in Section 2. In Section 3, new analytical approximations for the radial distribution of the spectral power of absorbed radiation are suggested and discussed. Application of the results to calculation of differential absorption of thermal radiation in a diesel fuel droplet is discussed in Section 4. The main results of the paper are summarised in Section 5.

2. Basic equations

We assume that the illumination of the droplet by the external radiation is spherically symmetric and the angular distribution of this radiation is known, the droplet's own thermal radiation can be ignored, and the droplet's shape can be approximated by that of a sphere. In this case the radiation transfer equation in the droplet can be presented as (Özişik, 1973; Siegel and Howell, 1992):

$$\mu \frac{\partial I_\lambda}{\partial r} + \frac{1-\mu^2}{r}\frac{\partial I_\lambda}{\partial \mu} + a_\lambda I_\lambda = 0, \tag{1}$$

where $I_\lambda(r,\mu)$ is the spectral radiation intensity at a given point integrated along the azimuthal angle, a_λ is the liquid absorption coefficient, r is the distance from the droplet centre, $\mu = \cos\theta$ (θ is measured from the r-direction).

The boundary conditions for $I_\lambda(r,\mu)$ can be presented as

$$\left.\begin{aligned} &I_\lambda(0,-\mu) = I_\lambda(0,\mu),\\ &I_\lambda(r_d,-\mu) = R(n,\mu)I_\lambda(r_d,\mu)\\ &\qquad + [1 - R(1/n,-\mu')]n^2 I_\lambda^{\text{ext}}(-\mu'), \end{aligned}\right\} \tag{2}$$

where $\mu' = \sqrt{1-n^2(1-\mu^2)}$, R is the reflection coefficient (Born and Wolf, 1975), n is the index of refraction of liquid, I_λ^{ext} is the spectral intensity of external radiation. Note that $R(1/n,-\mu') = R(n,\mu)$. The assumption that the illumination of the droplet by the external radiation is spherically symmetric does not imply the isotropy of the distribution of thermal radiation inside the droplet except in its centre.

The assumption that the droplet's thermal radiation can be ignored is justified in the case of fuel droplet heating by surrounding gas in diesel engines, where the fuel droplet temperature is much lower than the gas temperature. In the general case Eq. (1) would have to include the terms describing the sources of radiation inside the droplet (Dombrovsky, 2000). The effect of external radiation is accounted for by the boundary conditions (2).

The first boundary condition in (2) is the symmetry condition at the droplet centre. The second boundary condition in (2) indicates that the value of $I_\lambda(r_d,-\mu)$ at the surface of the droplet is the sum of the intensity of the reflected radiation (the first term in the right hand side of this equation) and the intensity of the refracted radiation (the second term in the right hand side of this equation).

The radiation power absorbed per unit volume of the droplet is determined as

$$P(r) = \int_0^\infty p_\lambda(r)\,d\lambda, \tag{3}$$

where

$$p_\lambda(r) = a_\lambda I_\lambda^0(r), \quad I_\lambda^0(r) = \int_{-1}^{1} I_\lambda(r,\mu)\,d\mu,$$

$I_\lambda^0(r)$ is the spectral radiation power density.

Calculation of $P(r)$ based on Eqs. (1)–(3) is rather difficult. Following Dombrovsky (2000, 2002a) this problem is simplified using the MDP_0 approximation. In this approximation it is assumed that in the droplet core ($r \leqslant r_* \equiv r_d/n$) radiation intensity is constant in the angular ranges $-1 \leqslant \mu < 0$ and $0 < \mu \leqslant 1$. At the droplet periphery ($r_* < r \leqslant r_d$), however, constant values of the radiation intensity are assumed when $-1 \leqslant \mu < -\mu_*$ and $\mu_* < \mu \leqslant 1$, where $\mu_* = \sqrt{1-(r_*/r)^2}$. External radiation cannot penetrate into the droplet peripheral zone at $-\mu_* < \mu < \mu_*$. Numerical solution of Eq. (1) with boundary conditions (2) have shown that the MDP_0

adequately predicts the angular dependence of the radiation intensity (Dombrovsky, 2000). This approximation is based on the analysis of the following function:

$$g_0(r) = \begin{cases} I^0_\lambda(r), & r \leqslant r_*, \\ I^0_\lambda(r)/(1-\mu_*), & r_* < r \leqslant r_\mathrm{d}. \end{cases} \quad (4)$$

Integration of Eqs. (1) and (2) over μ in the ranges $-1 \leqslant \mu < 0$ and $0 < \mu \leqslant 1$ (droplet core) and in the ranges $-1 \leqslant \mu < -\mu_*$ and $\mu_* < \mu \leqslant 1$ (droplet periphery) leads to the following boundary-value problem for $g_0(r)$ (cf. Dombrovsky, 2000, 2002a):

$$\frac{1}{4r^2}\left(r^2 g_0'\right)' + \frac{C_1 - 1}{2r} g_0' = C_2 a_\lambda^2 g_0, \quad (5)$$

where

$$C_1 = \begin{cases} 1 & \text{when } r \leqslant r_*, \\ (1-\mu_*)/2\mu_* & \text{when } r_* < r \leqslant r_\mathrm{d}, \end{cases}$$

$$C_2 = \begin{cases} 1 & \text{when } r \leqslant r_*, \\ (1+\mu_*)^{-2} & \text{when } r_* < r \leqslant r_\mathrm{d}, \end{cases}$$

the differentiation is over r.

The boundary conditions for Eq. (5) are written as

$$\left.\begin{aligned} & g_0' = 0 && \text{when } r = 0, \\ & \tfrac{1+\mu_\mathrm{c}}{2} g_0' = \tfrac{2na_\lambda}{n^2+1}\left(n^2 I_\lambda^{0(\mathrm{ext})} - g_0\right) && \text{when } r = r_\mathrm{d}, \end{aligned}\right\} \quad (6)$$

where

$$\mu_\mathrm{c} = \sqrt{1-(1/n^2)}, \quad I_\lambda^{0(\mathrm{ext})} = \int_{-1}^{1} I_\lambda^{\mathrm{ext}}(\mu)\,\mathrm{d}\mu.$$

When deriving Eqs. (5) and (6) it was assumed that a_λ does not depend on r and the average values of $R(n,\mu)$ in the ranges $-1 \leqslant \mu < -\mu_\mathrm{c}$ and $\mu_\mathrm{c} < \mu \leqslant 1$ are equal to $R(n, 1)$.

The spectral power of the radiation absorbed per unit volume inside the droplet is determined as

$$p_\lambda(r) = a_\lambda[1 - \mu_* \Theta(r - r_*)]g_0(r), \quad (7)$$

where Θ is the Heaviside unit step function. The total power absorbed by the droplet can be calculated as

$$P_\mathrm{total} = 4\pi \int_0^{r_\mathrm{d}} P(r) r^2\,\mathrm{d}r, \quad P(r) = \int_0^\infty p_\lambda(r)\,\mathrm{d}\lambda. \quad (8)$$

P_total can be calculated without the solution of the boundary-value problem (5)–(6) if the following relation is used (Dombrovsky, 1996):

$$P_\mathrm{total} = \pi r_\mathrm{d}^2 \int_0^\infty Q_\mathrm{a} I_\lambda^{0(\mathrm{ext})}\,\mathrm{d}\lambda, \quad (9)$$

where the efficiency factor of absorption, introduced in the Mie theory (Bohren and Huffman, 1983), is approximated as (Dombrovsky, 2002b):

$$Q_\mathrm{a} = \frac{4n}{(n+1)^2}[1 - \exp(-2a_\lambda r_\mathrm{d})]. \quad (10)$$

Presentation of the results for the differential absorption of thermal radiation is simplified if the following normalised function is introduced:

$$w(r) = p_\lambda(r) \Bigg/ \left[\frac{3}{r_\mathrm{d}^3}\int_0^{r_\mathrm{d}} p_\lambda(r) r^2\,\mathrm{d}r\right]. \quad (11)$$

Eqs. (8)–(11) allow us to calculate the radiation power absorbed per unit volume inside the droplet as

$$P(r) = \frac{0.75}{r_\mathrm{d}} \int_0^\infty Q_\mathrm{a} w(r) I_\lambda^{0(\mathrm{ext})}\,\mathrm{d}\lambda. \quad (12)$$

If the external thermal radiation is that of a black body at temperature T_ext then $I_\lambda^{0(\mathrm{ext})} = 4\pi B_\lambda(T_\mathrm{ext})$ and Eq. (12) is rewritten as

$$P(r) = \frac{3\pi}{r_\mathrm{d}} \int_0^\infty Q_\mathrm{a} w(r) B_\lambda(T_\mathrm{ext})\,\mathrm{d}\lambda, \quad (13)$$

where B_λ is the Planck function (Dombrovsky et al., 2001).

As follows from Eq. (12), the problem of the approximate calculation of the radiation power absorbed per unit volume inside droplets reduces to the problem of finding an approximation for $w(r)$. This is discussed in the next section.

3. Approximations for *w*

When $r \leqslant r_*$ Eq. (5) is simplified to $\left(r^2 g_0'\right)' = 4r^2 a_\lambda^2 g_0$. The general analytical solution of this equation is

$$g_0 = \alpha \frac{\sinh(2\tau)}{\tau}, \quad (14)$$

where $\tau = a_\lambda r$. This solution could be used as the analytical approximation of g_0 for the whole droplet. In view of potential applications in modelling of droplet heating in diesel engines, however, we will look for an even simpler expression for g_0. The simplest non-trivial expression for g_0 satisfying the boundary condition at $r = 0$ can be presented as

$$g_0 = \alpha\tau^2 + \beta. \quad (15)$$

Integration of Eq. (5) from 0 to $\tau_* = a_\lambda r_*$ gives

$$\tau_* g_0'(\tau_*) = 4\int_0^{\tau_*} g_0 \tau^2\,\mathrm{d}\tau. \quad (16)$$

Eq. (16) allows us to find the relation between the coefficients α and β and simplify the expression for g_0 to

$$g_0 = \alpha\left(\tau^2 + \gamma\right), \quad (17)$$

where $\gamma = 1.5 - 0.6(\tau_0/n)^2$, $\tau_0 = a_\lambda r_\mathrm{d}$.

The combination of Eqs. (17) and (11) gives

$$w(\bar{r}) = \frac{[1 - \mu_* \Theta(\bar{r} - 1/n)](\bar{r}^2 + \bar{\gamma})}{\left[0.6\left(1-\mu_\mathrm{c}^5\right) - \mu_\mathrm{c}^3/n^2\right] + \bar{\gamma}\left(1-\mu_\mathrm{c}^3\right)}, \quad (18)$$

where $\bar{\gamma} = (1.5/\tau_0^2) - (0.6/n^2)$, $\bar{r} = r/r_\mathrm{d}$.

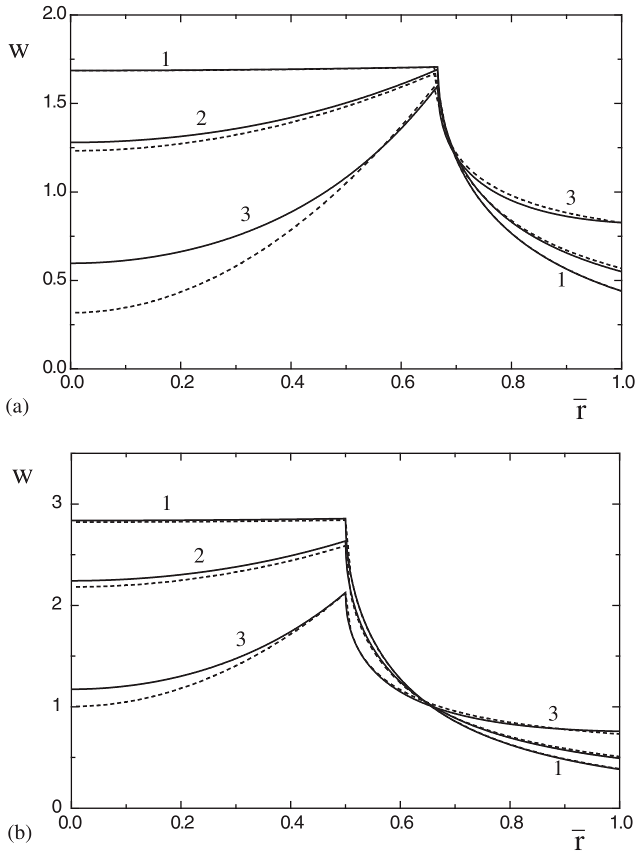


Fig. 1. Normalised profiles of thermal radiation power absorbed in a droplet versus dimensionless radius as predicted by the MDP_0 approximation (solid) and the simplified approximation (18) (dashed). Plots 'a' refer to $n = 1.5$, while plots 'b' refer to $n = 2$. Curves 1, 2 and 3 refer to optical thicknesses τ_0 equal to 0.2, 1 and 2 respectively.

Results of comparison of the values of $w(\bar{r})$ predicted by Eq. (18) and obtained from numerical solution of Eqs. (5)–(7) and (11) are presented in Fig. 1. Two values of n (1.5 and 2) and three values of τ_0 (0.2, 1 and 2) have been used for this comparison. As follows from this figure, the agreement between the results predicted by our approximation and the numerical solution is reasonably good.

The discontinuity of the slope shown in Fig. 1 takes place at $\bar{r} = 1/n$ ($1/n \approx 0.67$ in Fig. 1a; $1/n = 0.5$ in Fig. 1b). This can be given a qualitative explanation. All rays entering the droplet from outside, will concentrate in the cone with the half angle $\theta = \sin^{-1}(1/n)$ after refraction at the surface. The spheres of radii $\bar{r} = 1/n$ are the maximal spheres inside this cone. Note that in the MDP_0 approximation, on which the analysis of this paper is based, this discontinuity of slope is postulated. The rigorous solution does not predict any discontinuity, although the predicted slope of the curve changes rather rapidly in the vicinity of this point (Dombrovsky, 2000).

In the limit $\tau \to 0$, Eq. (15) gives $g_0 = \text{const}$. This coincides with the exact solution of the problem. When τ increases the deviation between the prediction of approximation (17) and the solution of the boundary-value problem (5)–(6) increases. As a result, the accuracy of our analytical approximation decreases, as shown in Fig. 1.

Approximation (17) and Eq. (18) are not valid for $\tau_0 > n\sqrt{2.5}$ when $g_0(0) < 0$. In this case we can approximate $p_\lambda(\tau)$ as

$$p_\lambda(\tau) = c\exp\left[-\xi(\tau_0 - \tau)\right], \tag{19}$$

where $\xi = 2/(1 + \mu_c)$.

Eq. (19) takes into account the refraction of radiation at the surface of the droplet, but does not describe adequately $p_\lambda(\tau)$ at $\tau < \tau_*$. In the latter range of τ, however, the values of $p_\lambda(\tau)$ are small for large τ_0. Substitution of Eq. (19) into Eq. (11) gives

$$w(\tau) = \frac{\xi^2\tau_0^3}{3}\,\frac{\exp\left[-\xi(\tau_0 - \tau)\right]}{\tau_0(\xi\tau_0 - 2) + (2/\xi)\left[1 - \exp\left(-\xi\tau_0\right)\right]}. \tag{20}$$

For large τ_0 Eq. (20) predicts physically correct absorption of thermal radiation in a thin layer near the droplet surface. The combination of Eqs. (10), (12), (18) and (20) can be used for practical estimates of the effects of differential absorption of thermal radiation in fuel droplets. The simplicity of these equations makes them particularly attractive for implementation into multidimensional CFD codes (Sazhina et al., 2000; Utyuzhnikov, 2002).

4. Applications

Let us now consider the problem of absorption of thermal radiation in a diesel fuel droplet. We assume that the spectrum of external radiation is that of a black body at temperatures $T_{ext} = 1500$ K and $T_{ext} = 2500$ K, which correspond to the typical temperatures in a combustion chamber of diesel engines (cf. Flynn et al., 1999). The spectrum of thermal radiation absorbed by fuel droplets in diesel engines can differ considerably from that of a black body if realistic temperature gradients inside the combustion chamber and radiative properties of combustion products are taken into account (Dombrovsky, 2002b). In our case, however, we are primarily interested in investigating the effect of the shift in the maximum of spectral density of radiation and not in quantitative estimate of the radiation flux. Droplet radii are assumed to be in the range from 10 to 50 μm (Pitcher et al., 1990; Comer et al., 1999).

A typical plot of the index of absorption of two types of diesel fuel versus λ are shown in Fig. 2 in the range 2 μm $< \lambda <$ 6 μm (Dombrovsky et al., 2003). Curve 1 refers to diesel fuel used in the analysis by Dombrovsky et al. (2001) (fuel 1); curve 2 refers to diesel fuel used in cars and described by Dombrovsky et al. (2003) (fuel 2). Note that Dombrovsky et al. (2003) described four dif-

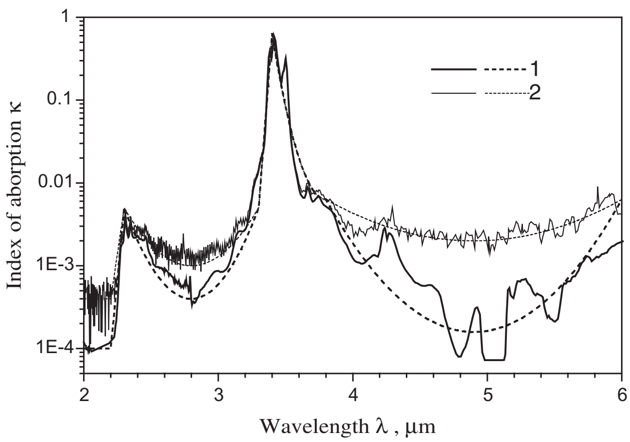


Fig. 2. Experimentally observed spectral index of absorption of two types of diesel fuels (solid curves). Curve 1 refers to diesel fuel studied by Dombrovsky et al. (2001); curve 2 refers to diesel fuel studied by Dombrovsky et al. (2003) (typical fuel used in cars). Approximation of these curves by Eq. (21) are shown by the corresponding dashed curves.

ferent types of diesel fuels: those used in cars and off-road vehicles, and the same fuels, for which the aging process was simulated by prolonged boiling. The spectral properties of all these types of fuel turned out to be rather similar. The index of absorption in the range 0.5 μm $< \lambda <$ 1.1 μm is not shown in this figure, but was used in the calculations. In the same figure approximations of these plots are shown. These approximations are given by the following function (Dombrovsky et al., 2002):

$$\kappa = 10^{\zeta}, \tag{21}$$

where

$$\zeta = -6.4 \quad \text{when } 0.5 \leqslant \lambda \leqslant 1,$$

$$\zeta = (6.4 - b_1)(\lambda - 1)^2 - 6.4 \quad \text{when } 1 \leqslant \lambda \leqslant 2,$$

$$\zeta = -b_1 \quad \text{when } 2 < \lambda \leqslant 2.2,$$

$$\zeta = 10(b_1 - 2.3)(\lambda - 2.2) - 3.4 \quad \text{when } 2.2 < \lambda \leqslant 2.3,$$

$$\zeta = 4(b_2 - 2.3)(\lambda - 2.8)^2 - b_2 \quad \text{when } 2.3 < \lambda \leqslant 3.3,$$

$$\zeta = 21(\lambda - 3.3)^2 - 2.3 \quad \text{when } 3.3 < \lambda \leqslant 3.4,$$

$$\zeta = 12.5(\lambda - 3.8)^2 - 2.2 \quad \text{when } 3.4 < \lambda \leqslant 3.8,$$

$$\zeta = (b_3 - 2.2)[(\lambda - 4.9)/1.1)]^2 - b_3 \quad \text{when } 3.8 < \lambda \leqslant 6,$$

$b_1 = 4$, $b_2 = 3.4$ and $b_3 = 3.8$ for fuel 1; $b_1 = 3.4$, $b_2 = 3$ and $b_3 = 2.7$ for fuel 2. λ is measured in μm.

Analytical presentation of $\kappa(\lambda)$ is convenient for practical calculations. Absorption bands for various hydrocarbon fuels practically coincide. This allows us to use approximation (21) for these fuels after simple adjustments to several coefficients (Dombrovsky et al., 2002). Note that the experimental data in the range 1.1 μm $< \lambda <$ 2 μm are not available to the authors and the corresponding approximation of κ in this range might not be reliable. This, however, is not expected to influence our conclusions. The spectral energy of thermal radiation at $\lambda <$ 0.5 μm and $\lambda >$ 6 μm is small in diesel engines environment and is ignored in our calculations (Mengüç et al., 1985).

The refractive index of liquid fuel $n(\lambda)$ is calculated from the measured index of absorption $\kappa(\lambda)$ using the subtractive Kramers–Krönig analysis (Ahrenkiel, 1971; Dombrovsky et al., 2003). The results of our calculations have been approximated by the following analytical formula (Dombrovsky et al., 2002):

$$n = n_0 + 0.02 \frac{\lambda - \lambda_m}{(\lambda - \lambda_m)^2 + 0.001}, \tag{22}$$

where $n_0 = 1.46$, $\lambda_m = 3.4$ μm, λ is the wavelength in μm.

Since the absorption spectra of various types of diesel fuels differ one from another only in the ranges of semi-transparency, the dependence of n on λ is expected to be practically the same for all types of diesel fuels. Hence, Eq. (22) is applicable for all types of diesel fuel.

For practical calculations the dependence of n on λ can sometimes be ignored, and n can be put equal to 1.46 (Dombrovsky et al., 2003). This will be discussed later in the paper.

Results of calculations of Q_a based on Eqs. (10) and (22) and the Mie theory are shown in Figs. 3 and 4. The Mie calculations were based on the algorithm described by Dombrovsky (1996). In these calculations the measured values of $\kappa(\lambda)$ (Dombrovsky et al., 2001, 2003) and calculated values of $n(\lambda)$ in the range 0.5 μm $\leqslant \lambda \leqslant$ 6 μm were used. Fuels 1 and 2, droplet radii 10 and 50 μm were used in the calculations. As follows from these figures, approximations (10), (21) and (22) are reasonable for practical applications for both fuels and for both droplet radii.

The normalised profiles of thermal radiation power absorbed in droplets are shown in Figs. 5 and 6 for both fuels. These profiles were calculated using the following equation:

$$W(\bar{r}) = \int_{\lambda_1}^{\lambda_2} w(\bar{r}) Q_a B_\lambda(T_{ext})\,d\lambda \Big/ \int_{\lambda_1}^{\lambda_2} Q_a B_\lambda(T_{ext})\,d\lambda. \tag{23}$$

It was assumed that $\lambda_1 = 0.5$ μm, $\lambda_2 = 6$ μm, and n was calculated from Eq. (22). Eq. (21) was used for $\kappa(\lambda)$ for both fuels. As follows from these figures, the analytical approximations (18) and (20) lead to results close to those predicted by more complicated calculations based on the solution of the boundary-value problem in the MDP_0 approximation. Note that $W(\bar{r})$ remains almost constant in the droplet core. This function achieves its minimal value in the peripheral zone. Finally, strong

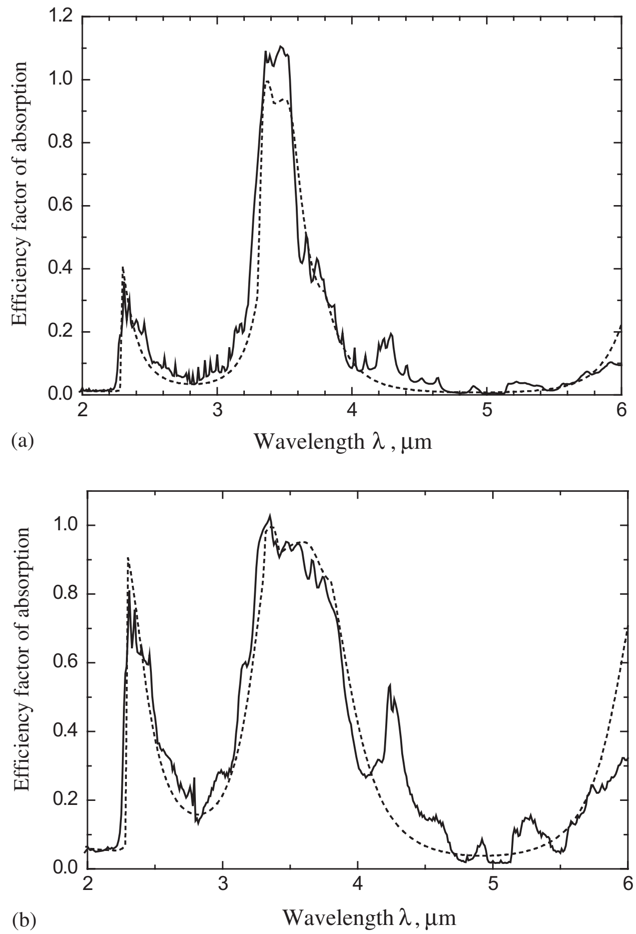


Fig. 3. Efficiency factor of absorption of diesel fuel 1 droplets with radii 10 μm (a) and 50 μm (b). Solid curves refer to rigorous calculations based on the Mie theory; dashed curves refer to the results predicted by approximations (10), (21) and (22).

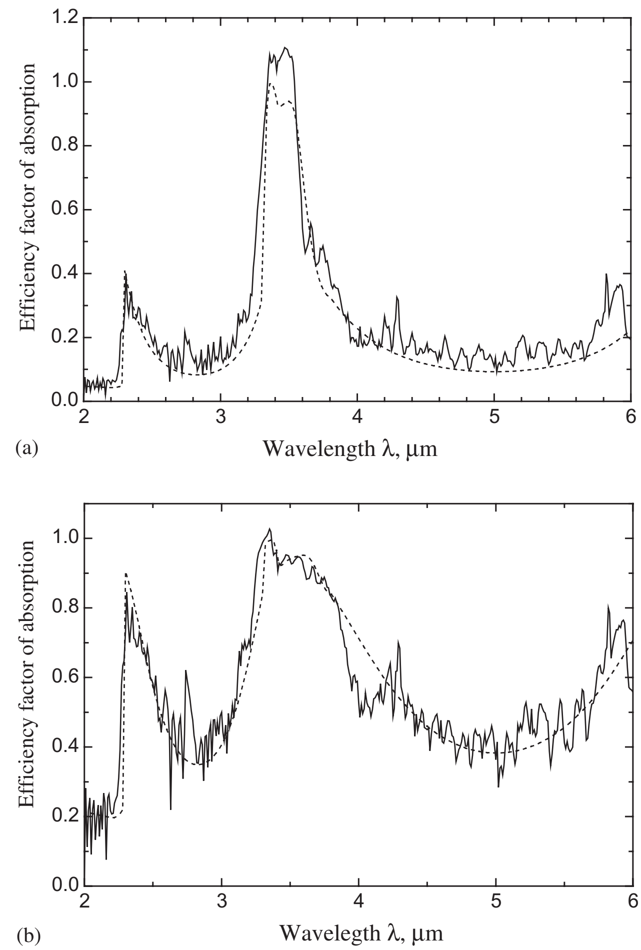


Fig. 4. The same as Fig. 3 but for diesel fuel 2.

absorption is predicted near the droplet surface. This behaviour of the function $W(\bar{r})$ could be related to the difference in absorption patterns of thermal radiation in the spectral ranges with low and high absorption. In the ranges of semi-transparency, radiation is absorbed mainly in the droplet core, while in the ranges of high absorption it is absorbed mainly near the droplet surface. As expected, the absorption near the surface is stronger and the relative thickness of the absorbing layer is smaller for larger droplets which have a larger optical thickness τ_0. Note that the influence of T_{ext} on function $W(\bar{r})$ is negligible.

For comparison, in Fig. 7 the plots similar to those in Fig. 5 are presented but for the case when the index of refraction is assumed constant and equal to 1.46. The main difference between the plots shown in Fig. 5 and those in Fig. 7 is that in the latter case there is a discontinuity in the slope of the curves at $\bar{r} = 1/n$, similar to the one shown in Fig. 1. The plots for fuel 2 (not shown) have the same property for constant n. Apart from this the difference between the curves for constant n and n defined by Eq. (22) is small. That means that in many practical applications the dependence of n on the wavelength can be ignored.

Harpole (1980) was probably the first to predict complex profiles of absorption of thermal radiation in water droplets. Absorption of thermal radiation in droplets core, however, was found to be noticeably weaker than that shown above for diesel fuels. This could be related to stronger absorption of thermal radiation by water compared with diesel fuel. Similarly to our results, Lage and Rangel (1993) predicted an almost constant absorption of thermal radiation in the n-decane droplet core. In contrast to our results, however, they predicted that most of the thermal radiation is absorbed near the droplet surface. This could be attributed to diesel fuel having a higher absorption than n-decane in semi-transparency spectral ranges (Dombrovsky et al., 2003). Also, no data for the absorption of n-decane at $\lambda < 2$ μm was available.

The new analytical approximation for the dimensionless thermal radiation power absorbed in liquid droplets could be used for various liquids in a rather

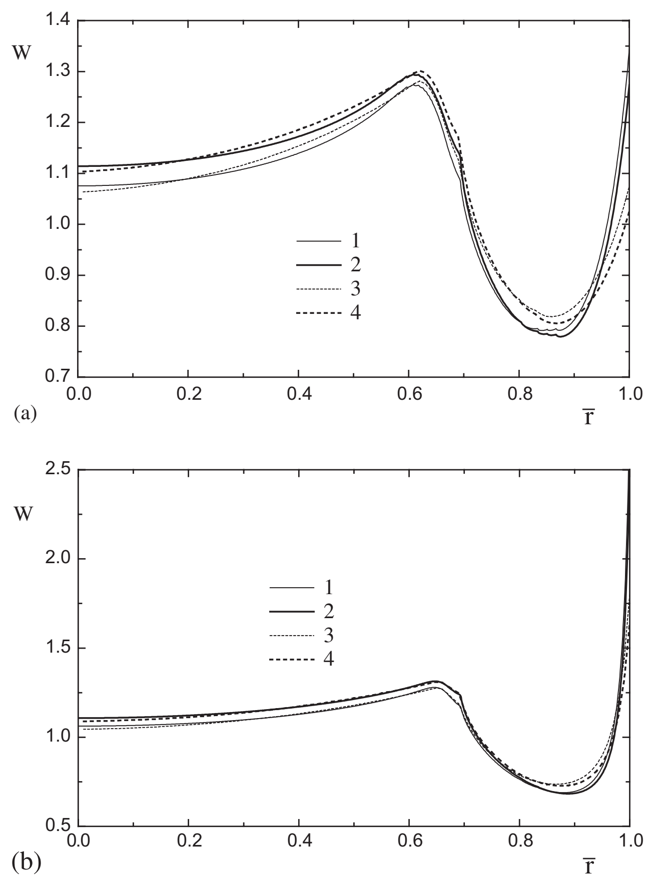


Fig. 5. Normalised profiles of thermal radiation power absorbed in diesel fuel 1 droplets with radii 10 μm (a) and 50 μm (b). Curves 1 and 2 refer to calculations based on the MDP_0 approximation and T_{ext} equal to 1500 and 2500 K respectively. Curves 3 and 4 refer to the results predicted by simplified approximations (18) and (20) and T_{ext} equal to 1500 and 2500 K respectively. The values of the fuel refractive index have been calculated from Eq. (22).

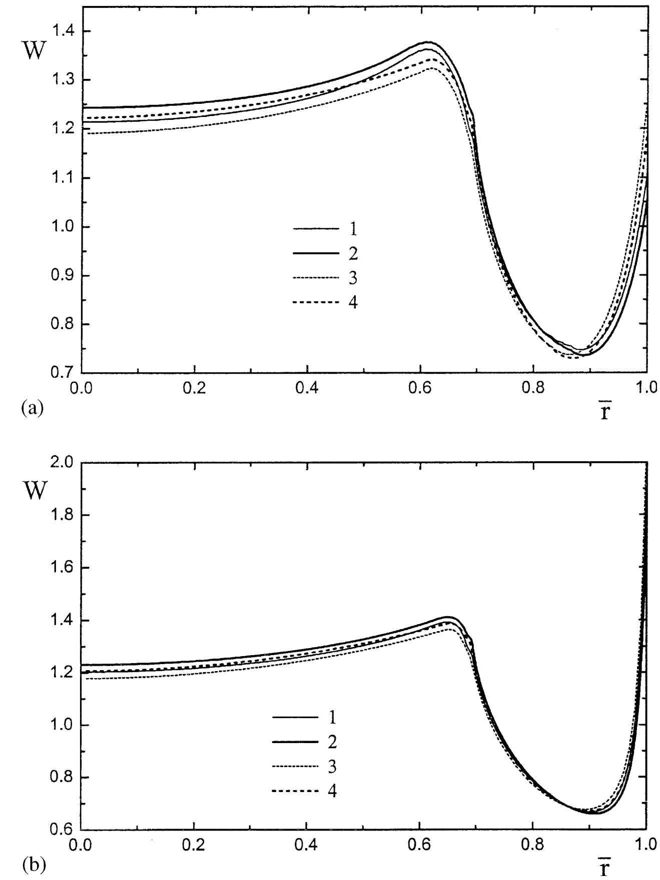


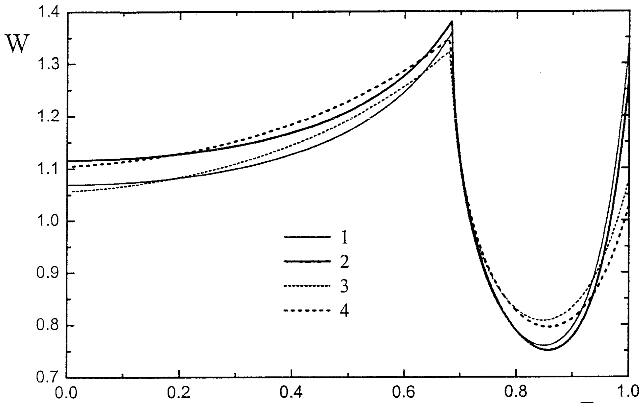
Fig. 6. The same as Fig. 5 but for diesel fuel 2.

wide range of parameters. It can allow us to calculate the profiles of thermal radiation power absorbed in droplets without using complex and time consuming algorithms. This is particularly important for potential applications in CFD codes (Gouesbet and Berlemont, 1999; Sazhina et al., 2000; Kang et al., 2001; Utyuzhnikov, 2002). If droplets other than those of diesel fuel are used, the approximate model presented in this paper might need to be modified. The general principles of the development of this modified model, however, are expected to be similar to the ones which we have described.

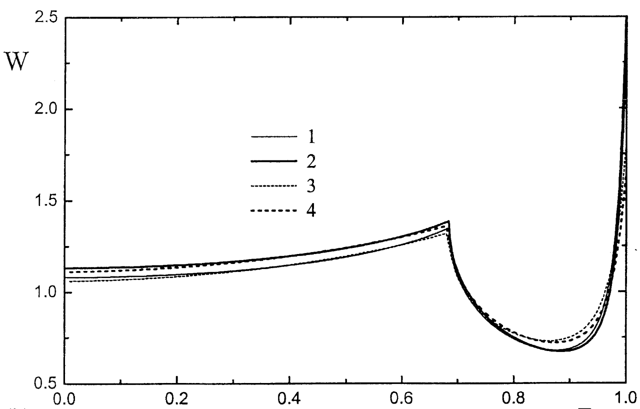
5. Conclusions

Distribution of absorption of thermal radiation in semi-transparent spherical droplets has been investigated assuming that the geometrical optics approximation is valid. Also, it has been assumed that the droplets are irradiated by spherically symmetric thermal radiation. The boundary-value problem for calculation of differential absorption of thermal radiation has been formulated based on the MDP_0 approximation. The solution of this problem has been supplemented by a simple analytical approximation for the normalised absorbed radiation power. This approximation has been combined with the analytical approximation for the efficiency factor of absorption, suggested earlier. It has been shown to retain the main features predicted by the numerical solution of the original equation used in the MDP_0 approximation.

The new approximation has been applied to the specific problem of differential absorption of thermal radiation in a diesel fuel droplet. Direct comparison of the results predicted by this approximation and those which follow from the original MDP_0 approximation has demonstrated the reasonable accuracy of the new approach for two types of diesel fuels and a wide range of droplet radii. It has been pointed out that the radial distribution of absorbed thermal radiation power is non-monotonic. Radiation power absorbed in the droplet core is shown to be rather large and almost homogeneous. It is also large in the vicinity of the droplet surface, but is minimal in the intermediate region. This



(a)



(b)

Fig. 7. The same as Fig. 5 but for the case when the refractive index of diesel fuel is assumed to be constant and equal to 1.46.

result has been related to the absorption spectra of diesel fuel.

The discontinuity of the slope of the normalised profile of thermal radiation absorbed in diesel droplets is predicted if the value of liquid fuel absorption coefficient is assumed constant. This discontinuity is removed when the dependence of the index of refraction on the wavelength is taken into account.

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